Separate Online Appendices with Supplemental Material for:

The Production Function for Housing: Evidence from France

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ABSTRACT: This document contains a set of appendices with supplemental material. Appendix F reports on our supplementary data sources. Appendix G, Appendix H, and Appendix I provide additional results for sections 5.1, 5.2, and 5.3 respectively. Appendix J provides evidence against two candidate explanations for the differences in capital elasticity that we estimate across urban areas. Appendix K provides details about how we implement our cost share correction. Appendix L and Appendix M report additional results for sections 5.4 and 5.5, respectively. Appendix N contains supplementary results for section 6. Finally, Appendix O provides an analysis of measurement error and smoothing.

Key words: housing, production function.

JEL classification: R14, R31, R32

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Appendix F. Additional data

Urban areas. We use the 1999 delineation of urban areas from the French statistical institute (INSEE).

Wages. We construct measures of wages for blue collar workers in the construction industry for all French urban areas from the French labour force administrative records (DADS - Déclarations Annuelles des Données Sociales).

Education. We construct measures of the share of population with a college or university degree for all French municipalities from the French census for 2006. We consider all higher education degrees that sanction two years of study or more after high school.

Income. Mean household income and its standard deviation by municipality can be constructed using information from each cadastral section (about 100 housing units on average) contained in the FILOCOM repository. This repository is managed by the *Direction Générale des Finances Publiques* of the French Ministry of Finance. It contains a record of all housing units and their occupants which they match to income tax records.

Soil variables. We use the European Soil Database compiled by the European Soil Data Centre. The data originally come as a raster data file with cells of 1 km per 1 km. We aggregated it at the level of each municipality and urban area. We refer to Combes, Duranton, Gobillon, and Roux (2010) for further description of these data.

Land use. We use information from the 2006 *Corine Land Cover* dataset to compute the share of agricultural and impervious land in each municipality. We compute the fraction of land that is built up in each municipality using information from *BD Topo* (version 2.1) from the French National Geographical Institute. This data set is originally produced using satellite imagery combined with the French land registry. It reports information for nearly all buildings in the country including their footprint, height, and use with an accuracy of one metre.

We also use the *Fichiers Fonciers du CEREMA* 3.0 for all dwellings to compute observed floor-to-area ratios in all municipalities. We select all residential houses (maisons) with three stories or less. We then allocate each of these houses to the parcel it sits on. Then, for each of the resulting parcels we compute the floor-to-area ratio by summing the built-up area of all dwellings on the parcels and divide by the area of the parcel. For each municipality, we keep the entire distribution of floor-to-area ratios for all parcels with a single-family home.

Values, rents, and mortgage rates for properties. Monthly rent data in euros per m^2 are from the *Clameur* consortium for 2012 (and published as "2013" rent data). The data are for 2,932 municipalities with population above 2,000 inhabitants in mainland France. The nearly 400,000 individual new leases or lease renewals that underlie the data are collected from members of the *Clameur* consortium, including financial institutions, large property management firms, associations of small property managers and real estate brokers, etc. A municipality is included only if 30 or more leases are observed. Direct rentals, which represent about 30% of the market are absent.
Property prices (which we refer to as 'values' in the main text to avoid any confusion with rental prices) are municipal indices constructed from the 2012 census of all transactions of non-new dwellings conducted by regional notary associations. For each quarter, the log of the price of houses is regressed on indicator variables for the construction period (before 1850, 1850-1913, 1914-1947, 1948-1959, 1960-1980, 1981-1991, after 1991) and a quarter indicator. We then use the output of this regression to compute a price prediction for a reference house in each municipality. We obtain an index for 26,972 municipalities in mainland France.
iAverage annual rates for mortgages are from l'Observatoire Crédit Logement / CSA, a consortium of the main French banks (which provide a joint-mortgage guarantee for a subset of properties akin to that of Fanny Mae in the US) and CSA, a market study firm with a long-running survey of housing finance in France.

Appendix G. Supplementary results for section 5.1: Technical checks

Table 9 duplicates table 2 using different smoothing bandwidths equal to a half, a quarter, and a tenth of the rule-of-thumb bandwidth we use in table 2 and other estimations. The results show that even strong under-smoothing barely affects the results.

Table 10 considers a broader support for K between the 3rd and 97th percentiles of all land values in the data instead of between the 10th and 90th percentile of parcel values in the distribution in table 2. This leads us to consider 94% of all land values instead of 75% in our base estimation. While we lose some precision in the estimates when introduce a quadratic term in panel (B), the results of panel (A) are similar to those of table 2.

Panels (A) and (B) of table 11 also duplicate table 2 but directly smooth the cost share $K^*/(K^* + R)$ instead of smoothing R prior to computing the cost share. Panels (C) and (D) of table 11 duplicate panels (A) and (B) of table 3 but smooth the cost share directly again. The results of the first two panels are similar to our base results but with mildly higher estimates of the capital elasticity by about 3 to 4 percentage points. The results of the last two panels are virtually undistinguishable from the corresponding results of table 3.

Appendix H. Supplementary results for section 5.2: Predicted values of K, R, and T

Table 12 report results experimenting with the set of demand-related factors. Panels (A) and (B) only include the urban area of a parcel to predict its price and capital investment. The results are qualitatively the same as those of the two panels of table 3. Despite the bluntness of this rudimentary exercise, we note a greater dispersion of the capital elasticity in panel (A) and more log concavity, especially for the lower deciles of parcel size in panel (B). Panels (C) and (D) rely again on the urban area of a parcel to predict its price and capital investment but condition out local wages in the construction industry from the estimated urban area fixed effects. With this specification, the differences in capital elasticity across parcel size deciles are minimal. Depending on the deciles, the production function of housing is either marginally log concave or marginally log convex. Panels (E) and (F) include urban area fixed effects, distance to the centre (with an effect specific to each urban area), income, and land-use variables among the demand determinants but do not condition out construction wages. Finally, panels (G) and (H) additionally condition out construction wages from urban area fixed effects and predict parcel size with demand-related factors.

Table 9: log housing production with different smoothing bandwidth, OLS by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): Bandwidth $=0.5\times$ rule-of-thumb bandwidth									
$\log(K)$	0.618^a	0.639^a	0.637^a	0.638^a	0.646^a	0.651^a	0.652^a	0.662^a	0.664^a
	(0.0011)	(0.00087)	(0.0011)	(0.0012)	(0.0014)	(0.0021)	(0.0022)	(0.0035)	(0.0038)
Panel (B): Bandwidth $=0.5\times$ rule-of-thumb bandwidth									
$\log(K)$	0.229^a	0.019	-0.107^b	0.089	-0.028	0.122	0.355^a	0.141	0.440^b
	(0.058)	(0.043)	(0.044)	(0.057)	(0.083)	(0.105)	(0.124)	(0.170)	(0.183)
$[\log(K)]^2$	0.016^a	0.026^a	0.031^a	0.023^a	0.028^a	0.022^a	0.013^b	0.022^a	0.009
	(0.0024)	(0.0018)	(0.0019)	(0.0024)	(0.0035)	(0.0044)	(0.0053)	(0.0071)	(0.0078)
Panel (C): Bandwidth $=0.25\times$ rule-of-thumb bandwidth									
$\log(K)$	0.616^a	0.638^a	0.635^a	0.640^a	0.650^a	0.653^a	0.650^a	0.663^a	0.666^a
	(0.0015)	(0.0011)	(0.0013)	(0.0020)	(0.0023)	(0.0026)	(0.0030)	(0.0046)	(0.0047)
Panel (D): Bandwidth $=0.25\times$ rule-of-thumb bandwidth									
$\log(K)$	0.258^a	-0.011	-0.076	0.184^b	-0.085	0.182	0.342^b	-0.100	0.548^b
	(0.085)	(0.049)	(0.063)	(0.088)	(0.113)	(0.152)	(0.159)	(0.233)	(0.230)
$[\log(K)]^2$	0.015^a	0.027^a	0.030^a	0.019^a	0.031^a	0.020^a	0.013^c	0.032^a	0.005
	(0.0036)	(0.0021)	(0.0027)	(0.0037)	(0.0048)	(0.0064)	(0.0067)	(0.010)	(0.010)
Panel (E): Bandwidth $=0.1\times$ rule-of-thumb bandwidth									
$\log(K)$	0.621^a	0.635^a	0.638^a	0.649^a	0.653^a	0.658^a	0.650^a	0.664^a	0.670^a
	(0.0024)	(0.0017)	(0.0032)	(0.0030)	(0.0038)	(0.0039)	(0.0064)	(0.0064)	(0.0071)
Panel (F): Bandwidth $=0.1\times$ rule-of-thumb bandwidth									
$\log(K)$	0.251^c	-0.027	-0.094	0.295	-0.101	0.147	0.183	-0.228	1.046^a
	(0.140)	(0.068)	(0.084)	(0.194)	(0.134)	(0.177)	(0.188)	(0.307)	(0.322)
$[\log(K)]^2$	0.016^a	0.028^a	0.031^a	0.015^c	0.032^a	0.022^a	0.020^b	0.038^a	-0.016
	(0.0059)	(0.0029)	(0.0036)	(0.0082)	(0.0057)	(0.0075)	(0.0080)	(0.013)	(0.014)

Notes: OLS regressions with a constant in all columns. 900 observations for each regression. The R^2 is 1.00 in all specifications. Bootstrapped standard errors in parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Table 10: log housing production, expanded support for R

Decile	1	2	3	4	5	6	7	8	9
Panel (A)									
$\log(K)$	0.633^a	0.647^a	0.650^a	0.647^a	0.648^a	0.654^a	0.654^a	0.662^a	0.663^a
	(0.015)	(0.016)	(0.017)	(0.017)	(0.016)	(0.015)	(0.015)	(0.016)	(0.016)
Panel (B)									
$\log(K)$	0.344	0.373	0.332	0.391	0.506	0.606	0.678	0.619	0.698
	(0.484)	(0.531)	(0.567)	(0.578)	(0.649)	(0.719)	(0.817)	(0.787)	(0.823)
$[\log(K)]^2$	0.012	0.011	0.013	0.010	0.006	0.002	-0.001	0.002	-0.001
	(0.021)	(0.023)	(0.024)	(0.025)	(0.028)	(0.030)	(0.035)	(0.034)	(0.035)

Notes: OLS regressions with a constant in all columns. Bootstrapped standard errors in parentheses. 900 observations for each regression. The R^2 is 1.00 in all specifications. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Table 11: log housing production with smoothed cost shares, by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): Observed data									
$\log(K)$	0.648^{a}	0.658^{a}	0.661^{a}	0.663^{a}	0.670^{a}	0.679^{a}	0.684^{a}	0.691^{a}	0.695^{a}
	(0.00077)	(0.00072)	(0.00071)	(0.00079)	(0.00080)	(0.0011)	(0.0011)	(0.0014)	(0.0016)
Panel (B): Observed data									
$\log(K)$	0.126^{a}	0.033	-0.021	0.085^{a}	0.118^{a}	0.198^{a}	0.333^{a}	0.243^{a}	0.321^{a}
	(0.038)	(0.025)	(0.025)	(0.031)	(0.036)	(0.046)	(0.068)	(0.083)	(0.082)
$[\log(K)]^2$	0.022^{a}	0.026^{a}	0.029^{a}	0.024^{a}	0.023^{a}	0.020^{a}	0.015^{a}	0.019^{a}	0.016^{a}
	(0.0016)	(0.0010)	(0.0011)	(0.0013)	(0.0015)	(0.0020)	(0.0029)	(0.0035)	(0.0035)
Panel (C): Predicted data									
$\log(K)$	0.653^{a}	0.654^{a}	0.655^{a}	0.658^{a}	0.663^{a}	0.671^{a}	0.675^{a}	0.680^{a}	0.686^{a}
	(0.0011)	(0.00065)	(0.00056)	(0.00069)	(0.00071)	(0.00092)	(0.0011)	(0.0014)	(0.0021)
Panel (D): Predicted data									
$\log(K)$	0.359^{a}	1.026^{a}	1.538^{a}	1.969^{a}	2.183^{a}	2.270^{a}	1.947^{a}	1.884^{a}	2.168^{a}
	(0.111)	(0.069)	(0.071)	(0.070)	(0.081)	(0.117)	(0.147)	(0.168)	(0.193)
$[\log(K)]^2$	0.012^{a}	-0.016^{a}	-0.037^{a}	-0.055^{a}	-0.064^{a}	-0.068^{a}	-0.054^{a}	-0.051^{a}	-0.063^{a}
	(0.0047)	(0.0029)	(0.0030)	(0.0030)	(0.0034)	(0.0049)	(0.0062)	(0.0071)	(0.0081)

Notes: OLS regressions with a constant in all columns. In panels (C) and (D), cost shares are predicted directly from the same demand-related factors used in table 3 to predict investment and parcel price separately. Bootstrapped standard errors in parentheses. 900 observations for each regression. The R^2 is 1.00 in all specifications. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Table 12: log housing production in urban areas obtained from predicted values, by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): Urban area fixed effects only									
log (K)	0.608^a	0.634^a	0.651^a	0.661^a	0.669^a	0.681^a	0.689^a	0.695^a	0.701^a
	(0.0022)	(0.0018)	(0.0014)	(0.0011)	(0.0014)	(0.0019)	(0.0022)	(0.0023)	(0.0028)
Panel (B): Urban area fixed effects only									
log (K)	5.741^a	5.389^a	3.508^a	2.682^a	2.904^a	2.957^a	2.927^a	2.921^a	3.207^a
	(0.331)	(0.239)	(0.236)	(0.169)	(0.224)	(0.285)	(0.332)	(0.362)	(0.439)
$[\log(K)]^2$	-0.217^a	-0.201^a	-0.121^a	-0.085^a	-0.094^a	-0.096^a	-0.095^a	-0.094^a	-0.106^a
	(0.014)	(0.010)	(0.010)	(0.0072)	(0.0095)	(0.012)	(0.014)	(0.015)	(0.019)
Panel (C): Urban area fixed effects net of construction wages									
log (K)	0.651^a	0.640^a	0.634^a	0.632^a	0.634^a	0.637^a	0.639^a	0.642^a	0.650^a
	(0.0034)	(0.00092)	(0.00086)	(0.0010)	(0.0013)	(0.0020)	(0.0027)	(0.0032)	(0.0054)
Panel (D): Urban area fixed effects net of construction wages									
log (K)	-1.132^b	-0.576^b	0.706^a	1.260^a	1.438^a	1.005^a	-0.122	-0.616	0.521
	(0.541)	(0.225)	(0.166)	(0.149)	(0.220)	(0.350)	(0.501)	(0.676)	(1.115)
$[\log(K)]^2$	0.075^a	0.051^a	-0.003	-0.027^a	-0.034^a	-0.016	0.032	0.053^c	0.005
	(0.0230)	(0.0095)	(0.0070)	(0.0063)	(0.0091)	(0.015)	(0.021)	(0.028)	(0.047)
Panel (E): Urban area fixed effects, distance effects, income, and land use									
log (K)	0.615^a	0.633^a	0.643^a	0.651^a	0.660^a	0.672^a	0.678^a	0.687^a	0.697^a
	(0.0011)	(0.00074)	(0.00074)	(0.00089)	(0.0012)	(0.0013)	(0.0016)	(0.0017)	(0.0021)
Panel (F): Urban area fixed effects, distance effects, income, and land use									
log (K)	3.795^a	3.532^a	3.508^a	3.708^a	3.944^a	4.091^a	4.191^a	4.117^a	4.034^a
	(0.087)	(0.071)	(0.064)	(0.067)	(0.085)	(0.093)	(0.108)	(0.126)	(0.159)
$[\log(K)]^2$	-0.134^a	-0.122^a	-0.121^a	-0.129^a	-0.139^a	-0.144^a	-0.148^a	-0.145^a	-0.141^a
	(0.0037)	(0.0030)	(0.0027)	(0.0029)	(0.0036)	(0.0039)	(0.0046)	(0.0054)	(0.0067)
Panel (G): ——— net of construction wages with predicted T									
log (K)	0.638^a	0.648^a	0.653^a	0.649^a	0.653^a	0.657^a	0.656^a	0.674^a	0.685^a
	(0.0020)	(0.0014)	(0.0018)	(0.0020)	(0.0015)	(0.0023)	(0.0029)	(0.0029)	(0.0030)
Panel (H): ——— net of construction wages with predicted T									
log (K)	-0.767^a	1.192^a	2.857^a	3.295^a	3.185^a	2.510^a	2.876^a	2.236^a	1.551^a
	(0.291)	(0.240)	(0.277)	(0.290)	(0.238)	(0.260)	(0.317)	(0.369)	(0.403)
$[\log(K)]^2$	0.059^a	-0.023^b	-0.093^a	-0.112^a	-0.107^a	-0.078^a	-0.094^a	-0.066^a	-0.037^b
	(0.012)	(0.010)	(0.012)	(0.012)	(0.010)	(0.011)	(0.013)	(0.016)	(0.017)

Notes: OLS regressions with a constant in all columns. In all panels (E)-(H), distance to the centre is urban-area specific; income variables are log mean municipal income, log standard error of income, and share of population with a university degree; geology variables are ruggedness, soil erodability, soil hydrogeological class, dominant parent material for two main classes of (lighter) soils; land use variables are three land use variables share of built-up land, share of urbanized land, and share of agricultural land. 900 observations for each regression. The R^2 is 1.00 in all specifications. Robust standard errors in parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Table 13: log housing production, by centiles of distance to the centre

Centiles	All urban areas	0-20	20-40	40-60	60-80	80-100
Panel (A): Observed data						
$\log(K)$	0.645^a	0.640^a	0.616^a	0.640^a	0.664^a	0.684^a
	(0.00087)	(0.0024)	(0.0015)	(0.0017)	(0.0017)	(0.0029)
Panel (B): Observed data						
$\log(K)$	0.086^a	-0.106^a	0.106^a	-0.116^a	-0.517^a	-0.509^a
	(0.030)	(0.076)	(0.051)	(0.066)	(0.064)	(0.099)
$[\log(K)]^2$	0.024^a	0.031^a	0.021^a	0.032^a	0.050^a	0.051^a
	(0.0013)	(0.0032)	(0.0022)	(0.0028)	(0.0027)	(0.0042)
Panel (C): Predicted data						
$\log(K)$	0.661^a	0.644^a	0.634^a	0.659^a	0.673^a	0.692^a
	(0.00062)	(0.0017)	(0.0016)	(0.0014)	(0.0070)	(0.0022)
Panel (D): Predicted data						
$\log(K)$	1.618^a	1.071^a	0.791^a	0.660^a	-0.203	1.270^c
	(0.078)	(0.203)	(0.181)	(0.175)	(1.771)	(0.694)
$[\log(K)]^2$	-0.040^a	-0.018^b	-0.007	-0.00018	0.037	-0.025
	(0.0033)	(0.0085)	(0.0076)	(0.0074)	(0.075)	(0.029)

Notes: OLS regressions with parcel size decile fixed effects in all columns. Centiles of distances are computed relative to the maximum distance from the centre in each urban area. In panels (C) and (D), K and R are predicted as in panel A of table 3. Bootstrapped standard errors in parentheses. 8,100 observations for each regression. The R^2 is 1.00 in all specifications. a, b, c: significant at 1%, 5%, 10%.

Appendix I. Supplementary results for section 5.3: Differences across locations

Table 13 reports results for different bands of distance to the centre. For each new construction, we compute the distance between the centroid of its municipality and the centroid of its urban area. We then normalize this distance by the maximum centroid-to-centroid distance in the urban area. Finally, we partition new constructions by their quintiles in the distribution of relative distances.

Table 14 complements table 4 with separate regressions for each decile of parcel size and confirms its results.

In table 15, we assess the effects of our measure of construction costs on the main results of table 2. In the first two panel, we used residualized values of K after conditioning out local construction wages (measured for each urban area). In the last two panels, we use a blunter approach and directly deflate capital investment by construction wages.

Table 14: log housing production by size class of urban areas, by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): Urban areas with 0 to 50,000 inhabitants, observed values									
$\log(K)$	0.707^a	0.705^a	0.702^a	0.703^a	0.709^a	0.715^a	0.718^a	0.722^a	0.729^a
	(0.0018)	(0.0019)	(0.0021)	(0.0023)	(0.0025)	(0.0031)	(0.0039)	(0.0046)	(0.0052)
Panel (B): Urban areas with 50,000 to 100,000 inhabitants, observed values									
$\log(K)$	0.694^a	0.688^a	0.683^a	0.685^a	0.694^a	0.707^a	0.714^a	0.710^a	0.706^a
	(0.0020)	(0.0018)	(0.0020)	(0.0022)	(0.0025)	(0.0027)	(0.0034)	(0.0053)	(0.0053)
Panel (C): Urban areas with 100,000 to 200,000 inhabitants, observed values									
$\log(K)$	0.684^a	0.681^a	0.678^a	0.677^a	0.678^a	0.683^a	0.692^a	0.700^a	0.705^a
	(0.00150)	(0.0014)	(0.0014)	(0.0017)	(0.0022)	(0.0024)	(0.0026)	(0.0025)	(0.0027)
Panel (D): Urban areas with 200,000 to 500,000 inhabitants, observed values									
$\log(K)$	0.651^a	0.650^a	0.648^a	0.644^a	0.638^a	0.636^a	0.639^a	0.651^a	0.657^a
	(0.0015)	(0.0013)	(0.0013)	(0.0016)	(0.0019)	(0.0022)	(0.0028)	(0.0026)	(0.0031)
Panel (E): Urban areas with more than 500,000 inhabitants (except Paris), observed values									
$\log(K)$	0.621^a	0.604^a	0.586^a	0.571^a	0.567^a	0.566^a	0.559^a	0.554^a	0.547^a
	(0.0016)	(0.0015)	(0.0016)	(0.0018)	(0.0020)	(0.0026)	(0.0039)	(0.0042)	(0.0047)
Panel (F): Paris, observed values									
$\log(K)$	0.516^a	0.521^a	0.529^a	0.537^a	0.546^a	0.552^a	0.552^a	0.551^a	0.548^a
	(0.0025)	(0.0024)	(0.0025)	(0.0029)	(0.0032)	(0.0034)	(0.0041)	(0.0042)	(0.0062)

Notes: OLS regressions with a constant in all columns. Bootstrapped standard errors in parentheses. 900 observations for each regression. The R^2 is 1.00 in all specifications. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Table 15: log housing production taking out construction costs, by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): Observed data after conditioning out constructions wages									
$\log(K)$	0.626[a]	0.639[a]	0.640[a]	0.638[a]	0.641[a]	0.648[a]	0.652[a]	0.657[a]	0.659[a]
	(0.0011)	(0.00077)	(0.00083)	(0.0011)	(0.0012)	(0.0014)	(0.0017)	(0.0023)	(0.0027)
Panel (B): Observed data after conditioning out constructions wages									
$\log(K)$	-0.410[a]	-0.476[a]	-0.552[a]	-0.410[a]	-0.373[a]	-0.274[a]	-0.186[b]	-0.254[b]	-0.201
	(0.041)	(0.035)	(0.037)	(0.043)	(0.056)	(0.077)	(0.081)	(0.101)	(0.135)
$[\log(K)]^2$	0.044[a]	0.047[a]	0.050[a]	0.044[a]	0.043[a]	0.039[a]	0.035[a]	0.038[a]	0.036[a]
	(0.0018)	(0.0013)	(0.0015)	(0.0018)	(0.0024)	(0.0033)	(0.0034)	(0.0043)	(0.0057)
Panel (C): Observed data after deflating by constructions wages									
$\log(K)$	0.625[a]	0.639[a]	0.640[a]	0.638[a]	0.642[a]	0.649[a]	0.653[a]	0.658[a]	0.660[a]
	(0.0010)	(0.00080)	(0.00088)	(0.00097)	(0.0012)	(0.0017)	(0.0019)	(0.0022)	(0.0030)
Panel (D): Observed data after deflating by constructions wages									
$\log(K)$	-0.185[a]	-0.290[a]	-0.371[a]	-0.241[a]	-0.201[a]	-0.101	0.011	-0.050	-0.005
	(0.044)	(0.032)	(0.035)	(0.046)	(0.064)	(0.066)	(0.092)	(0.097)	(0.127)
$[\log(K)]^2$	0.034[a]	0.039[a]	0.043[a]	0.037[a]	0.035[a]	0.032[a]	0.027[a]	0.030[a]	0.028[a]
	(0.0019)	(0.0013)	(0.0015)	(0.0019)	(0.0027)	(0.0028)	(0.0039)	(0.0041)	(0.0054)

Notes: OLS regressions with a constant in all columns. In panels (A) and (B), in a first step log capital investment is regressed on log constructions wages to derive a predicted value for K. In panels (C) and (D), capital investment is directly deflated by construction wages (beyond the year effects that we also use to make R comparable across years). Bootstrapped standard errors in parentheses. 900 observations for each regression. The R^2 is 1.00 in all specifications. a, b, c: significant at 1%, 5%, 10%.

Appendix J. Dismissing two explanations for differences in capital elasticity across urban areas

We first show that differences in capital elasticities across size classes of urban areas are unlikely to be the result of a complementarity between land and capital. To do this, we can use expression (14) derived in section 6 for a constant-elasticity-of-substitution approximation of the housing production function. Following expression (14), the capital elasticity for a parcel of size T in city c is equal to:

$$\frac{\partial \log H(K_c^*, T_c)}{\partial \log K_c^*} = \frac{\alpha (K_c^*)^{1-1/\sigma}}{\alpha (K_c^*)^{1-1/\sigma} + (1-\alpha) T_c^{1-1/\sigma}} = \frac{1}{1 + \frac{1-\alpha}{\alpha} \left(\frac{K_c^*}{T_c}\right)^{1/\sigma - 1}}, \tag{J1}$$

where σ is the elasticity of substitution and α is the share parameter in the CES production function.

We consider Paris for which the capital elasticity estimated in table 4 is equal to 0.539 and small urban areas (with population below 50,000) for which the capital elasticity estimated in the same table is equal to 0.712. We also consider parcels of 1,000 m^2. This is close to mean parcel size across all urban areas, as reported in table 1. To avoid computing construction costs from too few parcels, we compute the average construction cost for parcels of areas between 920 and 1100 m^2. There are 1,405 of them with average parcel size of 1,002 m^2 in Paris and 5,065 of them with average parcel size of 1,001 m^2 in small urban areas. Mean construction costs are 137,225 euros in small urban areas and 164,432 euros in Paris. For Paris, equation (J1) implies:

$$\frac{1}{1 + \frac{1-\alpha}{\alpha} (164.1)^{1/\sigma - 1}} = 0.539\,. \tag{J2}$$

For small urban areas, the same expression implies:

$$\frac{1}{1 + \frac{1-\alpha}{\alpha} (137.1)^{1/\sigma - 1}} = 0.712\,. \tag{J3}$$

Simple algebra shows that $\sigma = 0.194$ would be needed to satisfy these two equations. This value of σ is only a fraction of what we estimate below and is inconsistent with the stability of the capital elasticity we estimate within each class of urban areas in table 4. Comparing small urban areas with large urban areas with population above 500,000 instead of Paris leads to an even smaller value of 0.125 for σ.

The differences in capital elasticity we estimate across urban areas are also unlikely to be caused by differences in construction costs. As mentioned in the main text, construction wages are 14.2% higher in Paris than in small urban areas with a population less than 50,000. Construction wages are also 5.8% higher in large urban areas (with population above 500,000, excluding Paris) than in small urban areas. These figures are arguably an upper bound for the difference in construction costs since price differences for materials are expected to be less.

As above, we can use a constant-elasticity-of-substitution approximation of the production function. With $H_c = A\left(\alpha K_c^{(\sigma-1)/\sigma} + (1-\alpha) T_c^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$ and the cost of capital equal to r_c in city c, profit maximization by house builders implies:

$$\frac{\partial H(K_c,T_c)}{\partial K_c} = \frac{H(K_c,T_c)^{1/\sigma}}{K_c^{1/\sigma}} = \frac{r_c}{P_c}, \tag{J4}$$

Then using this last expression, it is easy to show that:

$$\frac{\partial \log H(K_c,T_c)}{\partial \log K_c} = \frac{H(K_c,T_c)^{1/\sigma-1}}{K_c^{1/\sigma-1}} = \left(\frac{P_c}{r_c}\right)^{\sigma-1}, \tag{J5}$$

For a higher cost of capital in larger cities to explain a lower share of capital in construction costs as we observe in the data, we need $\sigma > 1$. Recall that table 4 reports that the cost share is 32.1% higher in small urban areas than in Paris (0.712 vs. 0.539) and 23.8% in small urban areas than in large urban areas (0.712 vs. 0.575). Then, even if we ignore the higher cost of housing in Paris and large urban areas relative to small urban areas, we need $\sigma = 3.10$ for a 14.2% difference in cost between Paris and small urban areas to explain a 32.1% difference in capital elasticity. We also need $\sigma = 4.79$ for a 5.8% difference in cost between large and small urban areas to explain a 23.8% difference in capital elasticity.

Appendix K. Supplementary results for section 5.3: Corrected cost shares

To compute the corrected cost shares described by equation (c8), we need empirical values for τ, the rate of depreciation of housing capital, $\delta = \frac{1}{1+r}$, the discount factor, and for each of the size classes of cities used in table 4, $V_1 / [P_1 H(K_1^*)]$, the ratio of housing value to housing rent.

Starting with the rate of depreciation of housing capital, we take an annual value of 1% for the entire country. In the French national accounts, housing depreciation can be computed as the difference between investment in housing and the increase in housing stocks. According to Commissariat Général au Développement Durable (2012), this difference in 2009 was about 15 bn euros, which corresponds to slightly less than 1% of GDP or just below 0.6% of the value of the stock. This is arguably a lower bound as much housing maintenance falls under home production and is not accounted for in national accounts.

For the discount rate, we compute it using $r = 4\%$ which corresponds to the average annual rate for mortgages in France during our study period according to Observatoire Crédit Logement / CSA.

To compute the ratio of property values to annual rents for each class of city size we proceed as follows. We use monthly rent and property price data for 2012 as described in Appendix F. Three caveats are worth keeping in mind: (i) rent data are for an average of all observed transactions by the data provider, (ii) they only cover municipalities with a population above 2,000, and (iii) property values are for a reference property computed from all transactions. We consider only the 1,938 municipalities in an urban area for which we observe a new construction during our period, rent data, and property price data. Our sample covers 85% of municipalities with a population above 5,000. Because some urban areas are much larger than others, we regress the ratio of property rents to values on the inverse hyperbolic sine of the distance between the centroid of a property's municipality and the centroid of the urban area (which corresponds to the centroid of the main municipality) allowing for a different coefficient for each urban area. See Combes, Duranton, and Gobillon (2019) for further discussion. We use the results of this estimation to compute a distance-corrected rent to value ratio for each municipality in the data. Finally, for each size class of urban area, we take the median value among all represented municipalities rather than the mean to avoid giving too much weight to a few outlier municipalities.

Appendix L. Supplementary results for section 5.4: Land use regulations

Table 16 re-estimates our base results with decile indicators for each quintiles of absolute and relative FAR stringency using either smoothed observed data or predicted data for R and K.

Table 17 duplicates table 16 but computes local floor-to-area ratios using only single-family homes built after 2000 instead of the entire stock.

Table 18 estimates our bases results separately for 2006 to 2011 and for 2012 because the planning regime changes in 2012.

Table 16: log housing production, land use regulations

Centiles	All urban areas	0-20	20-40	40-60	60-80	80-100
Panel (A): Floor-to-area ratio, observed data						
$\log(K)$	0.644^a	0.679^a	0.649^a	0.631^a	0.621^a	0.614^a
	(0.00079)	(0.0018)	(0.0016)	(0.0018)	(0.0018)	(0.0017)
Panel (B): Floor-to-area ratio, observed data						
$\log(K)$	0.081^a	-0.433^a	-0.510^a	-0.427^a	-0.580^a	-0.249^a
	(0.032)	(0.082)	(0.072)	(0.056)	(0.051)	(0.056)
$[\log(K)]^2$	0.024^a	0.047^a	0.049^a	0.045^a	0.050^a	0.036^a
	(0.0014)	(0.0035)	(0.0030)	(0.0024)	(0.0021)	(0.0023)
Panel (C): Floor-to-area ratio, predicted data						
$\log(K)$	0.659^a	0.705^a	0.665^a	0.646^a	0.634^a	0.616^a
	(0.00054)	(0.0013)	(0.0015)	(0.0012)	(0.0011)	(0.0012)
Panel (D): Floor-to-area ratio, predicted data						
$\log(K)$	1.621^a	1.800^a	1.376^a	1.034^a	0.940^a	0.612^a
	(0.077)	(0.223)	(0.199)	(0.154)	(0.190)	(0.158)
$[\log(K)]^2$	-0.041^a	-0.047^a	-0.030^a	-0.016^a	-0.013^a	0.00001
	(0.0032)	(0.0095)	(0.0084)	(0.0065)	(0.0080)	(0.0066)
Panel (E): Relative floor-to-area ratio, observed data						
$\log(K)$	0.644^a	0.615^a	0.636^a	0.646^a	0.651^a	0.664^a
	(0.00077)	(0.0023)	(0.0023)	(0.0015)	(0.0013)	(0.0015)
Panel (F): Relative floor-to-area ratio, observed data						
$\log(K)$	0.081^a	-0.251^a	0.302^a	0.177^a	0.149^a	-0.088^a
	(0.030)	(0.081)	(0.074)	(0.060)	(0.060)	(0.050)
$[\log(K)]^2$	0.024^a	0.036^a	0.014^a	0.020^a	0.021^a	0.032^a
	(0.0013)	(0.0034)	(0.0031)	(0.0025)	(0.0025)	(0.0021)
Panel (G): Relative floor-to-area ratio, predicted data						
$\log(K)$	0.659^a	0.620^a	0.644^a	0.657^a	0.669^a	0.683^a
	(0.00062)	(0.0015)	(0.0013)	(0.0012)	(0.00090)	(0.0011)
Panel (H): Relative floor-to-area ratio, predicted data						
$\log(K)$	1.621^a	0.534^a	0.458^a	0.117^a	0.135^a	0.237^a
	(0.089)	(0.135)	(0.121)	(0.094)	(0.010)	(0.098)
$[\log(K)]^2$	-0.041^a	0.004^a	0.008^a	0.023^a	0.023^a	0.019^a
	(0.0038)	(0.0057)	(0.0051)	(0.0040)	(0.0042)	(0.0041)

Notes: OLS regressions with parcel size decile fixed effects in all columns. In panels (A)-(D), centiles of floor-to-area ratio are computed using the 30th percentile of property level floor-to-area ratio of all existing houses in each municipality. In panels (E)-(H), centiles of relative floor-to-area ratio are computed by measuring for each new construction the centile of their floor-to-area ratio in their municipal distribution before dividing them into five quantiles from least binding to most binding. In panels (B), (D), (F) and (H), K and R are predicted as in panel A of table 3. Bootstrapped standard errors in parentheses. 8,100 observations for each regression. The R^2 is 1.00 in all specifications. a, b, c: significant at 1%, 5%, 10%.

Table 17: log housing production, land use regulations post 2000 data

Centiles	All urban areas	0-20	20-40	40-60	60-80	80-100
Panel (A): Floor-to-area ratio, observed data after 2000						
$\log(K)$	0.644^a	0.692^a	0.655^a	0.635^a	0.619^a	0.592^a
	(0.00084)	(0.0020)	(0.0019)	(0.0016)	(0.0016)	(0.0016)
Panel (B): Floor-to-area ratio, observed data after 2000						
$\log(K)$	0.081^a	-0.640^a	-0.869^a	-0.779^a	-0.771^a	-0.771^a
	(0.030)	(0.071)	(0.073)	(0.054)	(0.061)	(0.043)
$[\log(K)]^2$	0.024^a	0.056^a	0.064^a	0.060^a	0.058^a	0.057^a
	(0.0013)	(0.0030)	(0.0031)	(0.0023)	(0.0026)	(0.0018)
Panel (C): Floor-to-area ratio, predicted data after 2000						
$\log(K)$	0.659^a	0.717^a	0.655^a	0.646^a	0.627^a	0.586^a
	(0.00050)	(0.0014)	(0.0042)	(0.0070)	(0.0039)	(0.0012)
Panel (D): Floor-to-area ratio, predicted data after 2000						
$\log(K)$	1.622^a	1.727^a	-0.477^a	1.364^a	0.792^a	-0.529^a
	(0.077)	(0.206)	(0.664)	(1.646)	(0.723)	(0.151)
$[\log(K)]^2$	-0.041^a	-0.043^a	0.048^a	-0.030^a	-0.007^a	0.047^a
	(0.0032)	(0.0087)	(0.028)	(0.069)	(0.030)	(0.0063)
Panel (E): Relative floor-to-area ratio, observed data after 2000						
$\log(K)$	0.644^a	0.611^a	0.633^a	0.646^a	0.653^a	0.674^a
	(0.00070)	(0.0022)	(0.0018)	(0.0017)	(0.0014)	(0.0013)
Panel (F): Relative floor-to-area ratio, observed data after 2000						
$\log(K)$	0.081^a	-0.133^a	0.364^a	0.214^a	-0.013	-0.160^a
	(0.036)	(0.076)	(0.072)	(0.060)	(0.054)	(0.049)
$[\log(K)]^2$	0.024^a	0.031^a	0.011^a	0.018^a	0.028^a	0.035^a
	(0.0015)	(0.0032)	(0.0030)	(0.0025)	(0.0023)	(0.0021)
Panel (G): Relative floor-to-area ratio, predicted data after 2000						
$\log(K)$	0.659^a	0.614^a	0.643^a	0.660^a	0.675^a	0.699^a
	(0.00055)	(0.0018)	(0.0013)	(0.0010)	(0.0011)	(0.0011)
Panel (H): Relative floor-to-area ratio, predicted data after 2000						
$\log(K)$	1.622^a	0.543^a	0.646^a	0.561^a	0.456^a	0.728^a
	(0.078)	(0.128)	(0.119)	(0.080)	(0.079)	(0.078)
$[\log(K)]^2$	-0.041^a	0.003^a	0.001	0.004^a	0.009^a	-0.001
	(0.0033)	(0.0054)	(0.0050)	(0.0034)	(0.0033)	(0.0033)

Notes: OLS regressions with parcel size decile fixed effects in all columns. In panels (A)-(D), centiles of floor-to-area ratio are computed using the 30th percentile of property level floor-to-area ratio in each municipality for single-family homes built after 2000. In panels (E)-(H), centiles of relative floor-to-area ratio are computed by measuring for each new construction the centile of their floor-to-area ratio in their municipal distribution before dividing them into five quantiles from least binding to most binding. In panels (B), (D), (F) and (H), K and R are predicted as in panel A of table 3. Bootstrapped standard errors in parentheses. 8,100 observations for each regression. The R^2 is 1.00 in all specifications. a, b, c: significant at 1%, 5%, 10%.

Table 18: 2006-2011 vs. 2012, by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): Observed data 2006-2011									
$\log(K)$	0.625^a	0.639^a	0.641^a	0.639^a	0.644^a	0.651^a	0.653^a	0.660^a	0.659^a
	(0.0014)	(0.0013)	(0.0014)	(0.0015)	(0.0017)	(0.0017)	(0.0022)	(0.0027)	(0.0029)
Panel (B): Observed data 2006-2011									
$\log(K)$	0.142^a	-0.032	-0.124^a	-0.024	0.017	0.177^a	0.339^a	0.263^b	0.155
	(0.042)	(0.031)	(0.037)	(0.050)	(0.058)	(0.080)	(0.105)	(0.127)	(0.138)
$[\log(K)]^2$	0.020^a	0.028^a	0.032^a	0.028^a	0.026^a	0.020^a	0.013^a	0.017^a	0.021^a
	(0.0018)	(0.0013)	(0.0016)	(0.0021)	(0.0024)	(0.0037)	(0.0044)	(0.0054)	(0.0058)
Panel (C): Observed data 2012									
$\log(K)$	0.635^a	0.638^a	0.641^a	0.643^a	0.648^a	0.652^a	0.657^a	0.664^a	0.672^a
	(0.0015)	(0.0013)	(0.0014)	(0.0015)	(0.0018)	(0.0023)	(0.0029)	(0.0035)	(0.0039)
Panel (D): Observed data 2012									
$\log(K)$	-0.158^b	-0.095	-0.074	-0.044	-0.062	-0.189	-0.259^c	-0.211	-0.116
	(0.079)	(0.065)	(0.061)	(0.068)	(0.083)	(0.115)	(0.143)	(0.147)	(0.178)
$[\log(K)]^2$	0.033^a	0.031^a	0.030^a	0.029^a	0.030^a	0.035^a	0.039^a	0.037^a	0.033^a
	(0.0034)	(0.0028)	(0.0026)	(0.0029)	(0.0035)	(0.0048)	(0.0060)	(0.0062)	(0.0075)

Notes: OLS regressions with a constant in all columns. In panels (A) and (B), data from 2006 to 2011. In panels (C) and (D), data from 2012. Bootstrapped standard errors in parentheses. 900 observations for each regression. The R^2 is 1.00 in all specifications. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Table 19: log housing production in urban areas at various degrees of completion, by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): Fully finished units, observed data									
$\log(K)$	0.645^a	0.654^a	0.656^a	0.653^a	0.657^a	0.662^a	0.667^a	0.669^a	0.661^a
	(0.0017)	(0.0015)	(0.0016)	(0.0018)	(0.0020)	(0.0024)	(0.0028)	(0.0033)	(0.0051)
Panel (B): Fully finished units, predicted data									
$\log(K)$	0.658^a	0.660^a	0.663^a	0.666^a	0.671^a	0.677^a	0.682^a	0.688^a	0.691^a
	(0.0017)	(0.0011)	(0.0011)	(0.0013)	(0.0016)	(0.0020)	(0.0026)	(0.0032)	(0.0039)
Panel (C): Ready to decorate, observed data									
$\log(K)$	0.620^a	0.633^a	0.636^a	0.636^a	0.639^a	0.647^a	0.652^a	0.654^a	0.661^a
	(0.0011)	(0.0010)	(0.0011)	(0.0011)	(0.0012)	(0.0015)	(0.0020)	(0.0022)	(0.0035)
Panel (D): Ready to decorate, predicted data									
$\log(K)$	0.644^a	0.646^a	0.647^a	0.649^a	0.654^a	0.661^a	0.667^a	0.669^a	0.677^a
	(0.0013)	(0.00081)	(0.00074)	(0.00071)	(0.00094)	(0.00132)	(0.00156)	(0.0019)	(0.0024)
Panel (E): Structure completed, observed data									
$\log(K)$	0.593^a	0.603^a	0.608^a	0.607^a	0.607^a	0.611^a	0.615^a	0.614^a	0.614^a
	(0.0030)	(0.0028)	(0.0031)	(0.0040)	(0.0043)	(0.0045)	(0.0053)	(0.0068)	(0.0090)
Panel (F): Structure completed, predicted data									
$\log(K)$	0.619^a	0.615^a	0.611^a	0.608^a	0.606^a	0.612^a	0.622^a	0.632^a	0.636^a
	(0.0047)	(0.0032)	(0.0025)	(0.0026)	(0.0035)	(0.0043)	(0.0052)	(0.0071)	(0.0081)

Notes: OLS regressions with a constant in all columns. In panels (B), (D), and (F), K and R are predicted as in panel A of table 3. Bootstrapped standard errors in parentheses. 900 observations for each regression. The R^2 is 1.00 in all specifications. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Appendix M. Supplementary results for section 5.5: Housing heterogeneity

Table 19 reports results for different levels of completion.

Table 20 reports result by occupational groups of buyers.

Table 20: log housing production in urban areas across owners' occupations, by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): Executives, observed data									
$\log(K)$	0.612^a	0.624^a	0.627^a	0.625^a	0.628^a	0.628^a	0.628^a	0.631^a	0.620^a
	(0.0020)	(0.0017)	(0.0017)	(0.0021)	(0.0026)	(0.0026)	(0.0032)	(0.0036)	(0.0062)
Panel (B): Executives, predicted data									
$\log(K)$	0.633^a	0.632^a	0.634^a	0.638^a	0.644^a	0.650^a	0.652^a	0.656^a	0.655^a
	(0.0019)	(0.0012)	(0.0012)	(0.0013)	(0.0016)	(0.0023)	(0.0032)	(0.0040)	(0.0050)
Panel (C): Intermediate occupations, observed data									
$\log(K)$	0.627^a	0.636^a	0.639^a	0.639^a	0.642^a	0.645^a	0.653^a	0.657^a	0.650^a
	(0.0033)	(0.0032)	(0.0035)	(0.0036)	(0.0040)	(0.0047)	(0.0050)	(0.0060)	(0.0087)
Panel (D): Intermediate occupations, predicted data									
$\log(K)$	0.652^a	0.650^a	0.647^a	0.645^a	0.647^a	0.651^a	0.658^a	0.663^a	0.666^a
	(0.0024)	(0.0017)	(0.0015)	(0.0018)	(0.0022)	(0.0027)	(0.0035)	(0.0039)	(0.0048)
Panel (E): Clerical and blue-collar workers, observed data									
$\log(K)$	0.641^a	0.645^a	0.647^a	0.649^a	0.654^a	0.658^a	0.663^a	0.669^a	0.674^a
	(0.0014)	(0.0011)	(0.0010)	(0.0012)	(0.0014)	(0.0016)	(0.0019)	(0.0020)	(0.0025)
Panel (F): Clerical and blue-collar workers, predicted data									
$\log(K)$	0.663^a	0.657^a	0.654^a	0.653^a	0.654^a	0.656^a	0.659^a	0.660^a	0.660^a
	(0.0020)	(0.0012)	(0.00090)	(0.00090)	(0.0012)	(0.0017)	(0.0023)	(0.0025)	(0.0029)

Notes: OLS regressions with a constant in all columns. In panels (B), (D), and (F), K and R are predicted as in panel (A) of table 3. Bootstrapped standard errors in parentheses. 900 observations for each regression. The R^2 is 1.00 in all specifications. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Appendix N. Supplementary results for section 6

Table 21 duplicates table 2 after fitting the data to a Cobb-Douglas function in panels (A) and (B), a CES function in panels (C) and (D), a second-order translog function in panels (E) and (F), and a third order translog in panels (G) and (H).

Table 22 duplicates table 3 in the same way using predicted values for housing capital and parcel price.

Table 21: log housing production fitting specific functional forms, by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): Cobb-Douglas									
$\log(K)$	0.634^a	0.634^a	0.634^a	0.634^a	0.634^a	0.634^a	0.634^a	0.634^a	0.634^a
	(0.00070)	(0.00070)	(0.00070)	(0.00070)	(0.00070)	(0.00070)	(0.00070)	(0.00070)	(0.00070)
Panel (B): Cobb-Douglas									
$\log(K)$	0.634^a	0.634^a	0.634^a	0.634^a	0.634^a	0.634^a	0.634^a	0.634^a	0.634^a
	(0.00070)	(0.00070)	(0.00070)	(0.00070)	(0.00070)	(0.00070)	(0.00070)	(0.00070)	(0.00070)
$[\log(K)]^2$	-1.4e-7	1.4e-7	1.1e-7	-0.18e-7	-0.57e-7	-1.2e-7	0.93e-7	1.9e-7	1.3e-7
	(1.5e-7)	(1.5e-7)	(1.6e-7)	(1.4e-7)	(1.7e-7)	(1.7e-7)	(1.5e-7)	(1.6e-7)	(1.7e-7)
Panel (C): CES									
$\log(K)$	0.638^a	0.636^a	0.634^a	0.633^a	0.632^a	0.631^a	0.630^a	0.630^a	0.629^a
	(0.0010)	(0.00080)	(0.00071)	(0.00070)	(0.00075)	(0.00081)	(0.00089)	(0.0010)	(0.0011)
Panel (D): CES									
$\log(K)$	0.563^a	0.561^a	0.559^a	0.557^a	0.556^a	0.555^a	0.554^a	0.554^a	0.553^a
	(0.012)	(0.012)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.014)
$[\log(K)]^2$	0.0032^a	0.0032^a	0.0032^a	0.0032^a	0.0032^a	0.0032^a	0.0032^a	0.0032^a	0.0032^a
	(0.00053)	(0.00053)	(0.00053)	(0.00053)	(0.00053)	(0.00054)	(0.00054)	(0.00054)	(0.00054)
Panel (E): Second-order translog									
$\log(K)$	0.628^a	0.633^a	0.637^a	0.641^a	0.643^a	0.646^a	0.648^a	0.650^a	0.651^a
	(0.0010)	(0.00079)	(0.00072)	(0.00075)	(0.00084)	(0.00094)	(0.0010)	(0.0011)	(0.0012)
Panel (F): Second-order translog									
$\log(K)$	-0.149^a	-0.144^a	-0.140^a	-0.136^a	-0.134^a	-0.131^a	-0.129^a	-0.127^a	-0.126^a
	(0.019)	(0.019)	(0.019)	(0.019)	(0.019)	(0.019)	(0.019)	(0.019)	(0.019)
$[\log(K)]^2$	0.033^a	0.033^a	0.033^a	0.033^a	0.033^a	0.033^a	0.033^a	0.033^a	0.033^a
	(0.00082)	(0.00082)	(0.00082)	(0.00082)	(0.00082)	(0.00082)	(0.00082)	(0.00082)	(0.00082)
Panel (G): Third-order translog									
$\log(K)$	0.633^a	0.635^a	0.638^a	0.641^a	0.645^a	0.648^a	0.652^a	0.655^a	0.658^a
	(0.00116)	(0.00082)	(0.00081)	(0.00078)	(0.00082)	(0.00099)	(0.0013)	(0.0016)	(0.0020)
Panel (H): Third-order translog									
$\log(K)$	-0.048	-0.032	-0.017	-0.005	0.006	0.015	0.024	0.033	0.040
	(0.037)	(0.024)	(0.021)	(0.024)	(0.030)	(0.035)	(0.041)	(0.046)	(0.051)
$[\log(K)]^2$	0.029^a	0.028^a	0.028^a	0.027^a	0.027^a	0.027^a	0.026^a	0.026^a	0.026^a
	(0.0016)	(0.0010)	(0.00087)	(0.0010)	(0.0012)	(0.0015)	(0.0017)	(0.0020)	(0.0022)

Notes: OLS regressions with a constant in all columns. 900 observations for each regression. The R^2 is 1.00 in all specifications. Bootstrapped standard errors in parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%. For the second-order translog, there is a single coefficient for all deciles of parcel size for the term in log K squared by definition.

Table 22: log housing production fitting specific functional forms and using predicted values, by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): Cobb-Douglas									
$\log(K)$	0.652^a	0.652^a	0.652^a	0.652^a	0.652^a	0.652^a	0.652^a	0.652^a	0.652^a
	(0.00044)	(0.00044)	(0.00044)	(0.00044)	(0.00044)	(0.00044)	(0.00044)	(0.00044)	(0.00044)
Panel (B): Cobb-Douglas									
$\log(K)$	0.652^a	0.652^a	0.652^a	0.652^a	0.652^a	0.652^a	0.652^a	0.652^a	0.652^a
	(0.00044)	(0.00044)	(0.00044)	(0.00044)	(0.00044)	(0.00044)	(0.00044)	(0.00044)	(0.00044)
$[\log(K)]^2$	3.8e-7	-3.4e-7	2.8e-7	-3.2e-7	7.6e-7	1.9e-7	1.5e-7	5.4e-7	1.8e-7
	(5.8e-7)	(6.0e-7)	(6.0e-7)	(6.3e-7)	(5.6e-7)	(5.7e-7)	(5.8e-7)	(5.6e-7)	(5.0e-7)
Panel (C): CES									
$\log(K)$	0.635^a	0.644^a	0.650^a	0.655^a	0.659^a	0.663^a	0.666^a	0.669^a	0.671^a
	(0.00091)	(0.00056)	(0.00044)	(0.00050)	(0.00062)	(0.00076)	(0.00089)	(0.0010)	(0.0011)
Panel (D): CES									
$\log(K)$	0.938^a	0.944^a	0.948^a	0.951^a	0.953^a	0.955^a	0.957^a	0.959^a	0.960^a
	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)
$[\log(K)]^2$	-0.013^a	-0.013^a	-0.013^a	-0.012^a	-0.012^a	-0.012^a	-0.012^a	-0.012^a	-0.012^a
	(0.00066)	(0.00065)	(0.00064)	(0.00063)	(0.00062)	(0.00062)	(0.00061)	(0.00060)	(0.00060)
Panel (E): Second-order translog									
$\log(K)$	0.634^a	0.643^a	0.649^a	0.654^a	0.659^a	0.662^a	0.666^a	0.668^a	0.671^a
	(0.00095)	(0.00060)	(0.00047)	(0.00052)	(0.00065)	(0.00079)	(0.00093)	(0.0011)	(0.0012)
Panel (F): Second-order translog									
$\log(K)$	1.593^a	1.602^a	1.608^a	1.613^a	1.618^a	1.621^a	1.624^a	1.627^a	1.630^a
	(0.054)	(0.055)	(0.055)	(0.055)	(0.055)	(0.055)	(0.055)	(0.055)	(0.055)
$[\log(K)]^2$	-0.041^a	-0.041^a	-0.041^a	-0.041^a	-0.041^a	-0.041^a	-0.041^a	-0.041^a	-0.041^a
	(0.0023)	(0.0023)	(0.0023)	(0.0023)	(0.0023)	(0.0023)	(0.0023)	(0.0023)	(0.0023)
Panel (G): Third-order translog									
$\log(K)$	0.646^a	0.645^a	0.649^a	0.654^a	0.659^a	0.665^a	0.672^a	0.678^a	0.684^a
	(0.00140)	(0.00056)	(0.00054)	(0.00061)	(0.00067)	(0.00080)	(0.0010)	(0.0013)	(0.0014)
Panel (H): Third-order translog									
$\log(K)$	0.579^a	1.126^a	1.538^a	1.868^a	2.144^a	2.382^a	2.591^a	2.777^a	2.945^a
	(0.130)	(0.077)	(0.054)	(0.060)	(0.080)	(0.102)	(0.122)	(0.141)	(0.159)
$[\log(K)]^2$	0.0028	-0.020^a	-0.038^a	-0.051^a	-0.063^a	-0.073^a	-0.081^a	-0.089^a	-0.096^a
	(0.0055)	(0.0033)	(0.0023)	(0.0025)	(0.0034)	(0.0043)	(0.0052)	(0.0060)	(0.0067)

Notes: OLS regressions with a constant in all columns. Capital and parcel price are predicted from demand-related factors as in table 3. Observed values of parcel size are used. 900 observations for each regression. The R^2 is 1.00 in all specifications. Bootstrapped standard errors in parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%. For the second-order translog, there is a single coefficient for all deciles of parcel size for the term in log K squared by definition.

Appendix O. More on measurement error and smoothing

A. Details about the regressions of table 6

We consider seven sets of estimators for our parameters of interest:

1. *Traditional regression.* Following much of the literature, we estimate a linear specification of the log of housing capital per square metre of land, $\log(K/T)$, on the log parcel value per square metre, $\log(R/T)$. As shown below, the estimated constant $\widehat{\alpha}$ of the regression is an estimator of $a = \sigma \log[\alpha/(1-\alpha)]$ and the estimated coefficient of the explanatory variable is an estimator of the elasticity of substitution $\widehat{\sigma}$. An estimator of parameter α can then be recovered using the formula: $\widehat{\alpha} = \exp(\widehat{a}/\widehat{\sigma}) / [1 + \exp(\widehat{a}/\widehat{\sigma})]$.

2. *Traditional approach with smoothing*. We estimate a linear regression of $\log(K/T)$ on a smoothed version of $\log(R/T)$ that is obtained by replacing R with its kernel estimator, a bivariate normal kernel with rule-of-thumb bandwidth as used in our base approach. As previously, an estimator of α can be derived from the coefficients of the regression.

3. EGS. As described in Appendix B, Epple, Gordon, and Sieg (2010) show that with competitive house builders facing a concave, constant returns-to-scale production function, there is a relationship between R/T, the parcel price per square meter, and $V/T = PH/T$, the housing value per square meter of land: $R/T = f(V/T)$. Thanks to the zero profit condition, we also have: $K/T = V/T - f(V/T)$. We can use these expressions to recover predictors of parcel price per square meter $\widehat{R/T}$ and housing capital per square meter of land $\widehat{K/T}$ after reconstructing the house value as $V = K + R$ and approximating the function $f(\cdot)$ with a polynomial expansion, which we choose to be of order 10. Following Ahlfeldt and McMillen (2020), we then estimate a linear specification of $\log\left(\widehat{K/T}\right)$ on $\log\left(\widehat{R/T}\right)$ as with the traditional regression. Using the same transformation as in 1. and 2., we end up with an estimated share parameter and an estimated elasticity of substitution for the corresponding CES production function.

4. EGS *with smoothing*. We follow the same approach as in 3. but use a smoothed version of R when reconstructing the value of houses.

5. *Cost share*. We obtain estimates of α and σ by minimising the difference between the cost shares computed from the data and the theoretical expression obtained for a CES production function in equation (14) using non-linear least squares.

6. *Cost share with smoothing*. This duplicates approach 5. except that we use the smoothed version of R when computing the cost shares from the data.

7. *Our approach*. It is similar to the cost share approach with smoothed parcel prices except that, instead of considering differences between the cost shares computed from the data and the theoretical cost shares for every observation, we consider values on a 300×300 grid, and we weight each point on the grid with the sum of kernel weights (since the predictor of parcel price is more accurate when there are more observations in the neighborhood). This leads to a regression with 90,000 observations.

Table 6 in the text reports the results of these estimations. We also assess the robustness of these estimation techniques below with Monte-Carlo simulations.

B. Estimating the traditional regression with heterogeneous coefficients

To capture the effects of factor heterogeneity, we extend the standard CES production function to heterogeneous coefficients:

$$H_i(K_i,T_i) = A_i \left[\alpha_i K_i^{\frac{\sigma_i - 1}{\sigma_i}} + (1-\alpha_i) T_i^{\frac{\sigma_i - 1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i - 1}}, \tag{01}$$

for any parcel i. Following a derivation analogous to that leading to equation (14), we obtain:

$$\frac{K_i}{K_i + R_i} = \frac{\alpha_i K_i^{\frac{\sigma_i - 1}{\sigma_i}}}{\alpha_i K_i^{\frac{\sigma_i - 1}{\sigma_i}} + (1-\alpha_i) T_i^{\frac{\sigma_i - 1}{\sigma_i}}}, \tag{02}$$

where K_i is the profit-maximising capital investment (which we do not star to ease notations). Simple algebra shows that this last equation is equivalent to:

$$\log\left(\frac{K_i}{T_i}\right) = a_i + \sigma_i \log\left(\frac{R_i}{T_i}\right), \tag{03}$$

where $a_i = \sigma_i \log\left(\frac{\alpha_i}{1-\alpha_i}\right)$ and $R_i = R\left(K_i, T_i, a_i, \sigma_i\right)$.

The observed price of parcels may contain some measurement error: $\log \widetilde{R}_i = \log R_i + \varepsilon_i$ where R_i is the 'true' price of parcel i and ε_i is an independent, identically distributed, and centered error term orthogonal to all other quantities. To simplify the notations further, we note $X_i^0 \equiv \log\left(\frac{R_i}{T_i}\right)$ the true value of log parcel price per square meter, $X_i \equiv \log\left(\frac{\widetilde{R}_i}{T_i}\right)$ its observed value, and $Y_i \equiv \log\left(\frac{K_i}{T_i}\right)$ the log of housing investment per square meter of land. In practice, the production function is not exactly CES as argued in section 6. Hence, there is a mis-specification term ψ_i that we add to the equilibrium relationship between capital per square meter and parcel price per square metre in equation (03). The specification brought to the data is given by:

$$Y_i = a_i + \sigma_i X_i + \psi_i - \sigma_i \varepsilon_i \,. \tag{04}$$

The literature usually estimates a version of this equation with homogenous coefficients a and σ. We assess what the presence of heterogeneous coefficients in the data generating process implies for the estimator of the main coefficient of interest, σ, the elasticity of substitution (a similar argumentation holds for a). We can rewrite equation (04) as:

$$Y_i = a + \sigma_i X_i + [\psi_i - \sigma_i \varepsilon_i + a_i - a + (\sigma_i - \sigma)\, X_i] = a + \sigma X_i + \eta_i \,, \tag{05}$$

with $\eta_i \equiv \psi_i - \sigma\varepsilon_i + a_i - a + (\sigma_i - \sigma)\, X_i$. Denote $X_\bullet$ and $Y_\bullet$ the sample means of, respectively, X_i and Y_i. According to the Frisch-Waugh theorem, the OLS estimator of σ is given by:

$$\begin{aligned} \widehat{\sigma} &= \left[\frac{1}{N}\sum_i (X_i - X_\bullet)'(X_i - X_\bullet)\right]^{-1} \frac{1}{N}\sum_i (X_i - X_\bullet)'(Y_i - Y_\bullet) \\ &\longrightarrow V(X_i)^{-1}\mathrm{cov}(X_i, Y_i)\,, \end{aligned} \tag{06}$$

where we have:

$$\begin{aligned} \mathrm{cov}(X_i, Y_i) &= \mathrm{cov}\left(X_i, a_i + \sigma_i X_i^0 + \psi_i\right) \\ &= \mathrm{cov}\left(X_i^0 + \varepsilon_i, a_i + \sigma_i X_i^0\right) \\ &= \mathrm{cov}\left(X_i^0, a_i + \sigma_i X_i^0\right) \end{aligned} \tag{07}$$

and the limit of the estimator $\widehat{\sigma}$ can be decomposed as:

$$\widehat{\sigma} \longrightarrow E\left(\sigma_i\right) + Unobs.\, Bias \;+\; M.\, Error\, Bias\,. \tag{08}$$

where *Unobs. Bias* and *M. Error Bias* capture the divergence from the average elasticity of substitution due to the heterogeneity of production function parameters and the existence of measurement errors, respectively. We have:

$$\begin{aligned} M.\, Error\, Bias &= \left[V(X_i)^{-1} - V(X_i^0)^{-1}\right]\mathrm{cov}\left(X_i^0, a_i + \sigma_i X_i^0\right) \\ &= -\frac{V(\varepsilon_i)}{V(X_i^0)\left[V(X_i^0) + V(\varepsilon_i)\right]}\mathrm{cov}\left(X_i^0, a_i + \sigma_i X_i^0\right)\,. \end{aligned} \tag{09}$$

When there is heterogeneity in the parameters, this term is hard to sign. When, production function parameters are constant ($a_i = a$ and $\sigma_i = \sigma$), we get:

$$M.\, Error\, Bias = -\sigma\frac{V(\varepsilon_i)}{V(X_0) + V(\varepsilon_i)} < 0\,. \tag{010}$$

The measurement error bias is negative and depends on the respective importance of the variances of measurement error and log parcel price per square metre, consistently with standard results in the literature. We also have:

$$Unobs. Bias = V\left(X_i^0\right)^{-1}\left[\operatorname{cov}\left(X_i^0, a_i + \sigma_i X_i^0\right) - E(\sigma_i)V\left(X_i^0\right)\right] . \tag{011}$$

This bias is different from zero because the log parcel price per square meter depends on a_i and σ_i. If these two terms were independent, the bias would be zero, as we can check that: $\operatorname{cov}\left(X_i^0, a_i + \sigma_i X_i^0\right) = \operatorname{cov}\left(X_i^0, \sigma_i X_i^0\right) = E(\sigma_i)V(X_i^0)$. When these two terms are not independent, the object to which $\widehat{\sigma}$ converges has no clear interpretation. Even in the absence of measurement errors, this object is the sum of the average elasticity of substitution and a deviation due to the correlation between individual-specific coefficients and the log of parcel price per square meter at the equilibrium.

C. Estimating the traditional regression with smoothing

We now assess the effect of replacing $\widetilde{R}_i$ on the right-hand side of specification (05) with a smoothed version of it. Consider for simplicity that the smoothing amounts to averaging parcel prices over the M constructions most similar to i (including i), denoted Θ_i, in the sense that for any $j \in \Theta_i$, we have K_j close to K_i and T_j close to T_i. The intuitions are similar when using a kernel for smoothing instead of considering the closest neighbors. In this case, we have:

$$X_j = X_j^0 + \varepsilon_j = X_i^0 + \zeta_{i,j} + \varepsilon_j , \tag{012}$$

with $\zeta_{i,j} \equiv X_j^0 - X_i^0$. We now consider $X_i^\bullet = \frac{1}{M}\sum_{j\in\Theta_i} X_j$ instead of X_i in the estimated specification:

$$X_i^\bullet = X_i^0 + \zeta_i^\bullet + \varepsilon_i^\bullet , \tag{013}$$

where $\zeta_i^\bullet = \frac{1}{M}\sum_{j\in\Theta_i}\zeta_{i,j}$ and $\varepsilon_i^\bullet = \frac{1}{M}\sum_{j\in\Theta_i}\varepsilon_j$. The specification can then be written as:

$$\begin{aligned} Y_i &= a_i + \sigma_i X_i^0 + \psi_i \\ &= a + \sigma X_i^0 + \left[\psi_i + a_i - a + (\sigma_i - \sigma)X_i^0\right] \\ &= a + \sigma X_i^\bullet + \left[\psi_i + a_i - a + (\sigma_i - \sigma)X_i^0 + \sigma\left(X_i^0 - X_i^\bullet\right)\right] \\ &= a + \sigma X_i^\bullet + \widetilde{\eta}_i , \end{aligned} \tag{014}$$

where the residual is now:

$$\widetilde{\eta}_i = \psi_i + a_i - a + (\sigma_i - \sigma)X_i^0 - \sigma\left(\varepsilon_i^\bullet + \zeta_i^\bullet\right) . \tag{015}$$

There are two differences relative to the situation without smoothing. We consider an average of measurement errors in the neighborhood of i instead of the measurement error of i and there is an additional term corresponding to the mis-specification introduced by the use of the average parcel price per square meter in the neighborhood of i instead of parcel price per square meter of i. We now have:

$$\begin{aligned}\widehat{\sigma} &= \left[\frac{1}{N}\sum_i \left(X_i^{\bullet} - X_{\bullet}^{\bullet}\right)' \left(X_i^{\bullet} - X_{\bullet}^{\bullet}\right)\right]^{-1} \frac{1}{N}\sum_i \left(X_i^{\bullet} - X_{\bullet}^{\bullet}\right)\left(Y_i - Y_{\bullet}\right) \\ &\longrightarrow V\left(X_i^{\bullet}\right)^{-1} \operatorname{cov}\left(X_i^{\bullet}, Y_i\right) , \end{aligned} \tag{016}$$

where $X_{\bullet}^{\bullet}$ is the sample average of $X_i^{\bullet}$. The covariance term verifies:

$$\begin{aligned}\operatorname{cov}\left(X_i^{\bullet}, Y_i\right) &= \operatorname{cov}\left(X_i^{\bullet}, a_i + \sigma_i X_i^0 + \psi_i\right) \\ &= \operatorname{cov}\left(X_i^0 + \zeta_i^{\bullet} + \varepsilon_i^{\bullet}, a_i + \sigma_i X_i^0\right) \\ &= \operatorname{cov}\left(X_i^0, a_i + \sigma_i X_i^0\right) + \operatorname{cov}\left(\zeta_i^{\bullet}, a_i + \sigma_i X_i^0\right) . \end{aligned} \tag{017}$$

This covariance involves the same term as without smoothing and an additional one coming from the approximation of X_i^0 with a local average. The limit of the estimator $\widehat{\sigma}$ can then be decomposed in the following way:

$$\widehat{\sigma} \longrightarrow E\left(\sigma_i\right) + Unobs.\ Bias\ +\ M.\ Error\ Bias\ +\ Smooth.\ Bias\,. \tag{018}$$

Compared to the case without smoothing, the bias due to measurement errors is modified but the one due to the heterogeneity in production function parameters remains the same. There is also an additional bias *Smooth. Bias* that comes from the smoothing of parcel prices per square metre. We now have:

$$\begin{aligned} M.\ Error\ Bias &= \left[V\left(X_i^0 + \varepsilon_i^{\bullet}\right)^{-1} - V\left(X_i^0\right)^{-1}\right] \operatorname{cov}\left(X_i^0, a_i + \sigma_i X_i^0\right) \\ &= -\frac{V\left(\varepsilon_i^{\bullet}\right)}{V\left(X_i^0\right)\left[V\left(X_i^0\right) + V\left(\varepsilon_i^{\bullet}\right)\right]} \operatorname{cov}\left(X_i^0, a_i + \sigma_i X_i^0\right) \\ &= -\frac{1}{M}\frac{V\left(\varepsilon_i\right)}{V\left(X_i^0\right)\left[V\left(X_i^0\right) + \frac{1}{M}V(\varepsilon_i)\right]} \operatorname{cov}\left(X_i^0, a_i + \sigma_i X_i^0\right) \\ &= -\left[\frac{V\left(X_i^0\right) + V(\varepsilon_i)}{MV\left(X_i^0\right) + V(\varepsilon_i)}\right] \frac{V(\varepsilon_i)\operatorname{cov}\left(X_i^0, a_i + \sigma_i X_i^0\right)}{V\left(X_i^0\right)\left[V\left(X_i^0\right) + V(\varepsilon_i)\right]} . \end{aligned} \tag{019}$$

Since the first right-hand side term in brackets is lower than 1, the bias due to measurement errors is smaller in absolute term than without smoothing and decreasing with the number of neighbors used in the approximation. In fact, we can also see that when $V(\varepsilon_i) \ll V\left(X_i^0\right)$ this bias is $1/M$ lower than that without smoothing. The bias due to heterogeneity in production function parameters is the same as before and is still given by:

$$Unobs.\ Bias = V\left(X_i^0\right)^{-1}\left[\operatorname{cov}\left(X_i^0, a_i + \sigma_i X_i^0\right) - E(\sigma_i)V\left(X_i^0\right)\right] . \tag{020}$$

Finally, the bias coming from smoothing is given by:

$$\begin{aligned} Smooth.\ Bias \quad = \quad & \left[V\left(X_i^0 + \zeta_i^\bullet + \varepsilon_i^\bullet\right)^{-1} - V\left(X_i^0 + \varepsilon_i^\bullet\right)^{-1}\right]\operatorname{cov}\left(X_i^0, a_i + \sigma_i X_i^0\right) \\ & + V\left(X_i^0 + \zeta_i^\bullet + \varepsilon_i^\bullet\right)^{-1}\operatorname{cov}\left(\zeta_i^\bullet, a_i + \sigma_i X_i^0\right) . \end{aligned} \tag{021}$$

It is the sum of two terms. The first one comes from the fact that smoothing affects the variance of the explanatory variable and the second one arises from the disruption of the covariance between the outcome and explanatory variable. We note that the first term is influenced by the variance of measurement errors. In fact, is it not possible to provide a linear decomposition that additively separates strictly all the effects.

It can be seen from our decomposition that smoothing with a large number of parcels (M large) decreases the bias due to measurements error but introduces a specification bias, as it usually does.

D. Monte-Carlo simulations with homogeneous coefficients

We now conduct Monte-Carlo simulations to compare the performances of the various approaches analyzed in section 6. More specifically, we assess their robustness to measurement error and the importance of the specification bias introduced by smoothing.

Consider that the data generating process is such that the production function is, for now, CES with constant coefficients. We can generate a parcel price for every observation in the data from the amount of housing capital and the area of the parcel, provided that values for α and σ are available. From equation (03), the price of parcels must verify:

$$R = K\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{K}{T}\right)^{\frac{\sigma-1}{\sigma}} . \tag{022}$$

In practice, we need values of α and σ that make empirical sense. The first set of values we use is from the cost share estimation in table 6, panel A, and column 5: $\alpha = 0.60$ and $\sigma = 1.05$. These values are close to those obtained from the CES approximation of our base approach. We then consider a range of alternative values for σ between 0.6 and 2. To retain empirically meaningful values of α, we impose set values of σ to our cost share non-linear regression to estimate the corresponding values for α. Cobb-Douglas is a special case here obtained with $\sigma = 1$ and $\alpha = 0.65$ above. In the end, we consider:

Table 23: Values of σ and α for Monte-Carlo simulations

σ	1.05	0.6	0.8	1	1.2	1.5	2
α	0.60	0.98	0.87	0.65	0.45	0.26	0.13

We also introduce some measurement error. The simulated price of parcel i verifies: $\log \widetilde{R} = \log R + \sqrt{\delta}.\sigma_V.u$ where $\log R$ is obtained from equation (022) and $\sqrt{\delta}.\sigma_V.u$ is the noise we introduce. The chosen value for parameter σ_V is the standard deviation of the difference between the observed and the smoothed prices of parcels (where smoothing is as per our base approach). In essence, we interpret the difference between the observed value and the prediction from smoothing as the measurement error in the data. This variation represents 91.6% the standard deviation of the logarithm of observed land prices. The term u is drawn in a centered normal law with unit variance. In results not reported below, we also experimented with a uniform law, also with unit variance (where we draw d uniformly from [0,1] and apply the transformation $u = \sqrt{12}.(d - .5)$ since the variance of a uniform law [0,1] is 1/12). The results are very similar. Finally, δ is a scale parameter that determines the importance of the variance of measurement errors as a fraction of σ_V^2, since we have $V\left(\sqrt{\delta}.\sigma_V.u\right) = \delta\sigma_V^2$. For this parameter, we consider five values: 0, 0.25, 0.5, 1, and 2.

E. Results for homogeneous coefficients

We first consider the case with no heterogeneity in the parameters of the CES production function. The estimated parameters are reported in table 24. They show that the traditional approach behaves well as long as there is no measurement error (panel A). As soon as measurement error is introduced in the data generating process (panels B to E), the estimated parameters are biased and the biases increase with the variance of the measurement error. Unsurprisingly the bias is towards zero for the elasticity of substitution which is directly estimated and more complex for the share parameter α which is inferred from the constant and the elasticity of substitution. Interestingly, the traditional approach with smoothed parcel prices behaves well even with heavy measurement error.

The patterns are similar for the adaptation of the approach of Epple *et al.* (2010) by Ahlfeldt and McMillen (2020) to estimate a CES production function. We observe the same difference between estimates using observed parcel prices and those using smoothed parcel prices, with albeit smaller biases for the estimates using observed parcel prices. We also note that the cost share approach yields estimated parameters that are only mildly biased, even when we use observed parcel prices which are infused with a lot of measurement error. Recall however that, with this approach, it is the dependent variable rather than an explanatory variable that is affected by the measurement error. The two other approaches, cost share with smoothed land prices and our approach, which involves a similar smoothing, behave very well. In absence of measurement error, the mis-specification bias introduced by smoothing is minimal while the bias caused by measurement errors remains small.

F. Monte-Carlo simulations with heterogeneous coefficients

In another set of simulations, we now consider that the parameters of the CES production function are heterogeneous. More specifically, we have α_i and σ_i drawn from uniform distribution such that they can at most be 10% above or below the values of parameters reported in table 23. That is, we have $\alpha_i = (1 + u_\alpha)\alpha$ and $\sigma_i = (1 + u_\sigma)\sigma$ where u_α and u_σ are drawn independently and uniformly from $[-0.1, 0.1]$. Note that we do not consider heterogeneous coefficients when $\alpha = 0.98$ (and $\sigma = 0.6$) since consistency requires the share parameter α_i to be less than one.

The results reported in table 25 are very close to those reported in table 24 with homogeneous coefficients. This implies that our estimations are only barely affected by the introduction of some heterogeneity in the parameters of the CES production function.

In conclusion, we find that all the approaches we consider behave well when there is no issue of measurement error. However, if one believes that there is measurement error on parcel prices, the traditional approach and EGS without smoothing should be avoided while the other approaches should be favoured.

Table 24: Monte-Carlo simulations with homogeneous parameters

Estimation Technique:		Traditional	Traditional smoothed	EGS AM	EGS AM smoothed	Cost share	Cost share smoothed	Our approach
Panel (A): $\delta = 0$								
σ 1.05	$\hat{\sigma}$	1.05	1.06	1.05	1.06	1.05	1.06	1.06
α 0.6	$\hat{\alpha}$	0.60	0.59	0.60	0.59	0.60	0.59	0.59
σ 0.6	$\hat{\sigma}$	0.60	0.62	0.58	0.60	0.60	0.62	0.61
α 0.98	$\hat{\alpha}$	0.98	0.97	0.98	0.98	0.98	0.97	0.98
σ 0.8	$\hat{\sigma}$	0.80	0.82	0.79	0.80	0.80	0.82	0.81
α 0.87	$\hat{\alpha}$	0.87	0.85	0.88	0.87	0.87	0.85	0.86
σ 1	$\hat{\sigma}$	1.00	1.01	1.00	1.01	1.00	1.01	1.01
α 0.65	$\hat{\alpha}$	0.65	0.63	0.65	0.63	0.65	0.63	0.64
σ 1.2	$\hat{\sigma}$	1.20	1.20	1.20	1.20	1.20	1.20	1.20
α 0.45	$\hat{\alpha}$	0.45	0.45	0.45	0.45	0.45	0.44	0.45
σ 1.5	$\hat{\sigma}$	1.50	1.47	1.51	1.48	1.50	1.49	1.48
α 0.26	$\hat{\alpha}$	0.26	0.27	0.26	0.27	0.26	0.27	0.27
σ 2	$\hat{\sigma}$	2.00	1.90	2.01	1.91	2.00	1.94	1.94
α 0.13	$\hat{\alpha}$	0.13	0.15	0.13	0.14	0.13	0.14	0.14
Panel (B): $\delta = 0.25$								
σ 1.05	$\hat{\sigma}$	0.83	1.06	0.97	1.05	1.05	1.06	1.06
α 0.6	$\hat{\alpha}$	0.84	0.58	0.69	0.58	0.60	0.58	0.58
σ 0.6	$\hat{\sigma}$	0.55	0.62	0.57	0.60	0.60	0.62	0.61
α 0.98	$\hat{\alpha}$	0.99	0.97	0.99	0.98	0.98	0.97	0.97
σ 0.8	$\hat{\sigma}$	0.69	0.82	0.74	0.80	0.80	0.82	0.81
α 0.87	$\hat{\alpha}$	0.95	0.85	0.92	0.87	0.87	0.84	0.85
σ 1	$\hat{\sigma}$	0.81	1.01	0.93	1.01	1.00	1.01	1.01
α 0.65	$\hat{\alpha}$	0.86	0.62	0.73	0.63	0.65	0.62	0.63
σ 1.2	$\hat{\sigma}$	0.89	1.20	1.09	1.20	1.19	1.20	1.20
α 0.45	$\hat{\alpha}$	0.78	0.44	0.56	0.44	0.45	0.43	0.44
σ 1.5	$\hat{\sigma}$	0.98	1.47	1.31	1.48	1.48	1.48	1.48
α 0.26	$\hat{\alpha}$	0.68	0.26	0.36	0.26	0.26	0.26	0.26
σ 2	$\hat{\sigma}$	1.03	1.89	1.64	1.91	1.96	1.93	1.94
α 0.13	$\hat{\alpha}$	0.62	0.14	0.20	0.14	0.13	0.13	0.13
Panel (C): $\delta = 0.5$								
σ 1.05	$\hat{\sigma}$	0.69	1.06	0.89	1.05	1.05	1.06	1.06
α 0.6	$\hat{\alpha}$	0.95	0.56	0.77	0.57	0.59	0.56	0.56
σ 0.6	$\hat{\sigma}$	0.51	0.62	0.56	0.60	0.61	0.63	0.62
α 0.98	$\hat{\alpha}$	1.00	0.97	0.99	0.98	0.98	0.97	0.97
σ 0.8	$\hat{\sigma}$	0.61	0.82	0.73	0.81	0.81	0.82	0.82
α 0.87	$\hat{\alpha}$	0.98	0.84	0.92	0.85	0.86	0.84	0.84
σ 1	$\hat{\sigma}$	0.68	1.01	0.85	1.00	1.00	1.02	1.01
α 0.65	$\hat{\alpha}$	0.95	0.61	0.81	0.63	0.64	0.61	0.61
σ 1.2	$\hat{\sigma}$	0.71	1.20	0.99	1.20	1.19	1.21	1.21
α 0.45	$\hat{\alpha}$	0.94	0.42	0.66	0.42	0.45	0.42	0.42
σ 1.5	$\hat{\sigma}$	0.73	1.47	1.16	1.49	1.48	1.49	1.49
α 0.26	$\hat{\alpha}$	0.93	0.25	0.47	0.25	0.27	0.25	0.25
σ 2	$\hat{\sigma}$	0.69	1.90	1.39	1.92	1.93	1.94	1.96
α 0.13	$\hat{\alpha}$	0.95	0.13	0.30	0.13	0.14	0.13	0.13

Table 24: Monte-Carlo simulations with homogeneous parameters (continued)

Estimation Technique:			Traditional	Traditional smoothed	EGS AM	EGS AM smoothed	Cost share	Cost share smoothed	Our approach
Panel (D): $\delta = 1$									
σ	1.05	$\hat{\sigma}$	0.51	1.06	0.79	1.04	1.05	1.06	1.06
α	0.60	$\hat{\alpha}$	1.00	0.55	0.87	0.56	0.59	0.54	0.55
σ	0.6	$\hat{\sigma}$	0.45	0.62	0.53	0.60	0.62	0.62	0.62
α	0.98	$\hat{\alpha}$	1.00	0.97	0.99	0.97	0.97	0.97	0.97
σ	0.8	$\hat{\sigma}$	0.50	0.82	0.67	0.81	0.81	0.82	0.82
α	0.87	$\hat{\alpha}$	1.00	0.83	0.96	0.84	0.85	0.83	0.83
σ	1	$\hat{\sigma}$	0.51	1.01	0.76	1.03	1.00	1.01	1.01
α	0.65	$\hat{\alpha}$	1.00	0.60	0.90	0.57	0.64	0.59	0.59
σ	1.2	$\hat{\sigma}$	0.51	1.20	0.85	1.19	1.18	1.20	1.20
α	0.45	$\hat{\alpha}$	1.00	0.41	0.82	0.41	0.45	0.40	0.40
σ	1.5	$\hat{\sigma}$	0.48	1.46	0.95	1.47	1.45	1.48	1.49
α	0.26	$\hat{\alpha}$	1.00	0.24	0.70	0.24	0.28	0.23	0.23
σ	2	$\hat{\sigma}$	0.42	1.88	1.06	1.91	1.86	1.92	1.95
α	0.13	$\hat{\alpha}$	1.00	0.13	0.57	0.12	0.15	0.12	0.12
Panel (E): $\delta = 2$									
σ	1.05	$\hat{\sigma}$	0.34	1.06	0.64	1.07	1.05	1.07	1.07
α	0.6	$\hat{\alpha}$	1.00	0.50	0.97	0.48	0.58	0.49	0.49
σ	0.6	$\hat{\sigma}$	0.36	0.62	0.48	0.60	0.64	0.62	0.62
α	0.98	$\hat{\alpha}$	1.00	0.96	1.00	0.97	0.97	0.96	0.96
σ	0.8	$\hat{\sigma}$	0.36	0.82	0.56	0.80	0.82	0.82	0.82
α	0.87	$\hat{\alpha}$	1.00	0.80	0.99	0.83	0.84	0.79	0.80
σ	1	$\hat{\sigma}$	0.35	1.01	0.62	1.01	1.00	1.02	1.02
α	0.65	$\hat{\alpha}$	1.00	0.55	0.97	0.55	0.63	0.54	0.54
σ	1.2	$\hat{\sigma}$	0.32	1.20	0.66	1.20	1.17	1.21	1.22
α	0.45	$\hat{\alpha}$	1.00	0.36	0.96	0.36	0.45	0.35	0.35
σ	1.5	$\hat{\sigma}$	0.28	1.46	0.70	1.46	1.41	1.49	1.51
α	0.26	$\hat{\alpha}$	1.00	0.21	0.94	0.21	0.28	0.20	0.19
σ	2	$\hat{\sigma}$	0.23	1.87	0.74	1.93	1.78	1.93	1.98
α	0.13	$\hat{\alpha}$	1.00	0.11	0.91	0.10	0.16	0.10	0.10

Table 25: Monte-Carlo simulations with heterogeneous parameters

Estimation Technique:			Traditional	Traditional smoothed	EGS AM	EGS AM smoothed	Cost share	Cost share smoothed	Our approach
Panel (A): $\delta = 0$									
σ	1.05	$\hat{\sigma}$	1.10	1.11	1.05	1.06	1.10	1.11	1.06
α	0.6	$\hat{\alpha}$	0.55	0.54	0.60	0.59	0.55	0.54	0.59
σ	0.8	$\hat{\sigma}$	0.87	0.89	0.79	0.80	0.87	0.89	0.81
α	0.87	$\hat{\alpha}$	0.88	0.86	0.88	0.87	0.88	0.86	0.86
σ	1	$\hat{\sigma}$	1.06	1.07	1.00	1.01	1.06	1.08	1.01
α	0.65	$\hat{\alpha}$	0.64	0.63	0.65	0.63	0.64	0.63	0.64
σ	1.2	$\hat{\sigma}$	1.27	1.27	1.20	1.20	1.27	1.27	1.20
α	0.45	$\hat{\alpha}$	0.44	0.44	0.45	0.45	0.44	0.44	0.45
σ	1.5	$\hat{\sigma}$	1.45	1.43	1.51	1.48	1.45	1.43	1.48
α	0.26	$\hat{\alpha}$	0.24	0.25	0.26	0.27	0.24	0.25	0.27
σ	2	$\hat{\sigma}$	2.09	1.97	2.01	1.91	2.09	2.02	1.94
α	0.13	$\hat{\alpha}$	0.13	0.15	0.13	0.14	0.13	0.14	0.14
Panel (B): $\delta = 0.25$									
σ	1.05	$\hat{\sigma}$	0.85	1.11	0.97	1.05	1.10	1.11	1.06
α	0.6	$\hat{\alpha}$	0.82	0.53	0.69	0.58	0.55	0.53	0.58
σ	0.8	$\hat{\sigma}$	0.74	0.89	0.74	0.80	0.87	0.89	0.81
α	0.87	$\hat{\alpha}$	0.95	0.86	0.92	0.87	0.87	0.86	0.85
σ	1	$\hat{\sigma}$	0.84	1.07	0.93	1.01	1.06	1.07	1.01
α	0.65	$\hat{\alpha}$	0.87	0.62	0.73	0.63	0.64	0.62	0.63
σ	1.2	$\hat{\sigma}$	0.92	1.27	1.09	1.20	1.27	1.27	1.20
α	0.45	$\hat{\alpha}$	0.78	0.43	0.56	0.44	0.44	0.43	0.44
σ	1.5	$\hat{\sigma}$	0.97	1.42	1.31	1.48	1.43	1.43	1.48
α	0.26	$\hat{\alpha}$	0.64	0.24	0.36	0.26	0.25	0.24	0.26
σ	2	$\hat{\sigma}$	1.03	1.97	1.64	1.91	2.05	2.02	1.94
α	0.13	$\hat{\alpha}$	0.65	0.14	0.20	0.14	0.13	0.13	0.13
Panel (C): $\delta = 0.5$									
σ	1.05	$\hat{\sigma}$	0.70	1.10	0.89	1.05	1.09	1.11	1.06
-	0.6	$\hat{\alpha}$	0.94	0.52	0.77	0.57	0.55	0.52	0.56
σ	0.8	$\hat{\sigma}$	0.64	0.89	0.73	0.81	0.88	0.89	0.82
α	0.87	$\hat{\alpha}$	0.98	0.85	0.92	0.85	0.87	0.85	0.84
σ	1	$\hat{\sigma}$	0.69	1.07	0.85	1.00	1.06	1.07	1.01
α	0.65	$\hat{\alpha}$	0.96	0.62	0.81	0.63	0.64	0.61	0.61
σ	1.2	$\hat{\sigma}$	0.72	1.26	0.99	1.20	1.26	1.27	1.21
α	0.45	$\hat{\alpha}$	0.94	0.42	0.66	0.42	0.44	0.42	0.42
σ	1.5	$\hat{\sigma}$	0.73	1.42	1.16	1.49	1.42	1.43	1.49
α	0.26	$\hat{\alpha}$	0.91	0.23	0.47	0.25	0.25	0.23	0.25
σ	2	$\hat{\sigma}$	0.68	1.97	1.39	1.92	2.02	2.03	1.96
α	0.13	$\hat{\alpha}$	0.96	0.14	0.30	0.13	0.14	0.13	0.13

Table 25: Monte-Carlo simulations with heterogeneous parameters (continued)

Estimation Technique:			Traditional	Traditional smoothed	EGS AM	EGS AM smoothed	Cost share	Cost share smoothed	Our approach
Panel (D): $\delta = 1$									
σ	1.05	$\hat{\sigma}$	0.51	1.10	0.79	1.04	1.09	1.11	1.06
α	0.6	$\hat{\alpha}$	1.00	0.50	0.87	0.56	0.55	0.49	0.55
σ	0.8	$\hat{\sigma}$	0.51	0.89	0.67	0.81	0.88	0.89	0.82
α	0.87	$\hat{\alpha}$	1.00	0.84	0.96	0.84	0.86	0.84	0.83
σ	0.65	$\hat{\sigma}$	0.51	1.07	0.76	1.03	1.06	1.08	1.01
α	-	$\hat{\alpha}$	1.00	0.59	0.90	0.57	0.63	0.58	0.59
σ	1.2	$\hat{\sigma}$	0.50	1.26	0.85	1.19	1.24	1.27	1.20
α	0.45	$\hat{\alpha}$	1.00	0.40	0.82	0.41	0.44	0.39	0.40
σ	1.5	$\hat{\sigma}$	0.49	1.41	0.95	1.47	1.39	1.42	1.49
α	0.26	$\hat{\alpha}$	1.00	0.22	0.70	0.24	0.26	0.22	0.23
σ	2	$\hat{\sigma}$	0.41	1.94	1.06	1.91	1.93	2.00	1.95
α	0.13	$\hat{\alpha}$	1.00	0.13	0.57	0.12	0.15	0.12	0.12
Panel (E): $\delta = 2$									
σ	1.05	$\hat{\sigma}$	0.33	1.10	0.64	1.07	1.09	1.11	1.07
α	0.6	$\hat{\alpha}$	1.00	0.45	0.97	0.48	0.54	0.44	0.49
σ	0.8	$\hat{\sigma}$	0.36	0.88	0.56	0.80	0.88	0.89	0.82
α	0.87	$\hat{\alpha}$	1.00	0.82	0.99	0.83	0.85	0.81	0.80
σ	1	$\hat{\sigma}$	0.34	1.06	0.62	1.01	1.05	1.07	1.02
α	0.65	$\hat{\alpha}$	1.00	0.56	0.97	0.55	0.63	0.55	0.54
σ	1.2	$\hat{\sigma}$	0.31	1.25	0.66	1.20	1.23	1.26	1.22
α	0.45	$\hat{\alpha}$	1.00	0.37	0.96	0.36	0.45	0.36	0.35
σ	1.5	$\hat{\sigma}$	0.29	1.40	0.70	1.46	1.36	1.42	1.51
α	0.26	$\hat{\alpha}$	1.00	0.19	0.94	0.21	0.27	0.19	0.19
σ	2	$\hat{\sigma}$	0.23	1.93	0.74	1.93	1.83	2.00	1.98
α	0.13	$\hat{\alpha}$	1.00	0.11	0.91	0.10	0.16	0.10	0.10

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